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LAMINAR FLOW HEAT TRANSFER IN THE ENTRANCE REGION OF SEMI-CIRCULAR TUBES WITH UNIFORM HEAT FLUX

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NOMENCLATURE

a, radius;
C_p, constant pressure specific heat;
$\bar{h}$, average heat-transfer coefficient, $q''/(\bar{T}_w - T_b)$;
k, fluid thermal conductivity;
Nu, Nusselt number, $\bar{h}(2a)/k$;
P, pressure;
q'', rate of heat transfer per unit area;
R, θ, Z, dimensional cylindrical coordinates;
r, θ, z, dimensionless cylindrical coordinates;
T, local fluid temperature;
T_b, local mixed-mean fluid temperature;
$\bar{T}_w$, average wall temperature for Case 1; average for Case 2 considers only circular part of tube;
$\bar{W}$, average axial velocity;
μ, viscosity;
ρ, density.

INTRODUCTION

LAMINAR flow heat transfer in ducts of various shapes has been catalogued by Shah and London [1] for different kinds of thermal boundary conditions. The thermally and hydrodynamically fully developed flow in semi-circular tubes has been considered by Eckert *et al.* [2] and Sparrow and Haji-Sheikh [3]. Published entrance region solutions, obtained by use of highly sophisticated mathematical and numerical techniques, are generally concerned with circular tubes, rectangular ducts, or parallel plates. However, the thermally developing flow region in semi-circular tubes does not appear to have been described in the literature. This solution is required for interpretation of analytical [4, 5] and experimental [6] results for heat transfer in circular tubes having twisted tape inserts. At very large tape pitch, the flow is similar to that in semi-circular tubes.

FORMULATION OF PROBLEM

The flow is assumed to be hydrodynamically fully developed and thermally developing so as to represent the behavior of fluids with high Prandtl number. Physical properties of the fluid are considered to be constant so that effects of free convection and temperature-dependent viscosity are neglected. When viscous dissipation is neglected, the governing differential equations can be written in dimensionless forms as follows:

Momentum equation:

$$\nabla^2 w = C \tag{1}$$

Energy equation:

$$\nabla^2 \phi = w\left(\frac{\partial \phi}{\partial z} + C_1\right) \tag{2}$$

where the dimensionless parameters are defined as follows:

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}, \quad r = R/a,$$

$$w = w/\bar{W}, \quad C = \frac{a^2}{\mu \bar{W}}\frac{\partial P}{\partial Z}, \quad \phi = \frac{T - T_b}{q''a/k},$$

$$z = \frac{Z}{aRePr}, \quad Re = \frac{\rho \bar{W} a}{\mu}, \quad Pr = \frac{\mu C_p}{k}.$$

Two different thermal boundary conditions are considered:

Case 1. Axially uniform heat flux and uniform wall temperature around the entire semi-circular tube at each axial location,

Case 2. Axially uniform heat flux and uniform wall temperature around the semi-circular portion and an insulated wall at the flat portion.

These two boundary conditions represent the limits of the fin effect of twisted tapes. The contrast C_1 in equation (2) thus becomes

$$C_1 = 2(\pi + 2)/\pi \quad \text{for Case 1}$$

$$C_1 = 2 \quad \text{for Case 2.}$$

The boundary conditions for equations (1) and (2) are

$$w = 0 \text{ at the wall for both cases}$$

$$\phi = \phi_w \text{ for Case 1}$$

$$\phi = \phi_w \text{ at } r = 1,\ 0 \leqslant \theta \leqslant \pi, \text{ and}$$

$$\frac{\partial \phi}{\partial \theta} = 0 \text{ at } \theta = 0 \text{ and } \pi,\ r \neq 1, \text{ for Case 2.}$$

NUMERICAL SOLUTION

Equation (1), Poisson's equation, was solved by the successive over-relaxation method. Equation (2), a parabolic equation, was solved by the explicit and stable DuFort–Frankel method.

Equation (2) was written in finite-difference form using non-equal axial step sizes, finer near the entrance, and equal radial and circumferential step sizes (20 × 20). The tube wall temperature was obtained from the boundary condition and the fluid temperature gradient at the wall. The Nusselt number was taken as the average of values obtained by (a) using the heat-transfer coefficient based on average wall temperature and (b) relating the heat input to the enthalpy rise of the fluid. Stable computational behavior was observed throughout. Details of the numerical solution procedures are given in [6].

The present numerical solution for the velocity distribution agrees exactly with the closed form analytical solution given in [3]. In accordance with the assumptions made in this analysis, these velocity profiles are valid throughout the entrance region for both thermal boundary conditions.

Figure 1 shows the temperature distribution for Case 1 at $\theta = \pi/2$ for various axial locations. At the entrance of the duct the fluid temperature is quite uniform. Since the wall temperature is assumed circumferentially uniform, the fluid temperature is parabolic further downstream. This

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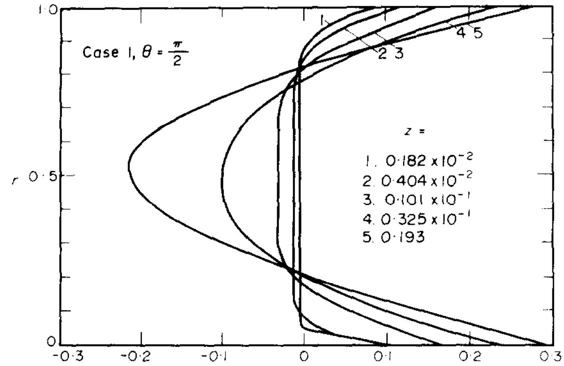

FIG. 1. Temperature profile development along the tube for Case 1.

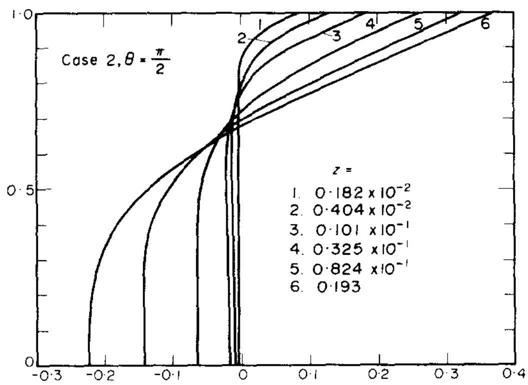

FIG. 2. Temperature profile development along the tube for Case 2.

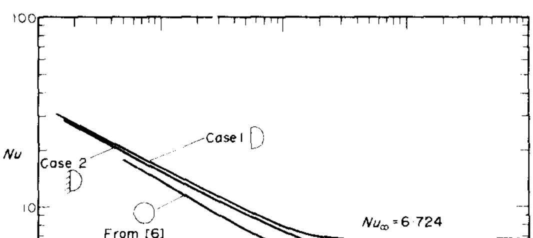

FIG. 3. Nusselt number in thermal entrance of semi-circular and circular tubes.

developing process is similar to thermally developing flow in circular tubes.

The temperature development in the entrance region at $\theta = \pi/2$ is demonstrated in Fig. 2 for the case of an insulated wall at the straight section of a semi-circular duct (Case 2). The temperature gradients at $r = 0$ are zero for no heat transfer across the wall. Once again, the temperature profiles are more uniform at the tube entrance than further downstream.

The final heat-transfer results are presented in Fig. 3. As for circular tubes, the Nusselt numbers in the entrance region are well above the asymptotic values. Heat-transfer coefficients for both cases are nearly identical at the tube inlet. Somewhere downstream, the Nusselt numbers for Case 1 become higher than the Nusselt numbers for Case 2; this is due to the larger heat-transfer area of Case 1. An interesting comparison of the present solutions with the heat-transfer results given in [6] for circular tubes is also illustrated in Fig. 3. It is seen that the Nusselt numbers for semi-circular tubes are higher than those for circular tubes.

The asymptotic values, Nu_∞, are in good agreement with results reported in the literature, after those results are converted to a similar basis (similar definitions of heat-transfer coefficient and diameter).* The comparison is given in the following table:

	Case 1	Case 2
Present solution	6·724	5·172
Sparrow and Haji-Sheikh [3]	6·692	—
Date [5]	6·599	5·188

The present analytical solution for semi-circular tubes will be utilized as a theoretical basis for experimental investigation of heat transfer augmentation by means of twisted tape inserts in circular tubes.

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REFERENCES

1. R. K. Shah and A. L. London, Thermal boundary conditions for laminar duct flow forced convection heat transfer, *J. Heat Transfer* **96**, 159 (1974).
2. E. R. G. Eckert, T. F. Irvine and J. T. Yen, Local laminar heat transfer in wedge-shaped passages, *Trans. Am. Soc. Mech. Engrs* **80**, 1433 (1958).
3. E. M. Sparrow and A. Haji-Sheikh, Flow and heat transfer in ducts of arbitrary shape with arbitrary thermal boundary conditions, *J. Heat Transfer* **88C**, 351 (1966).
4. A. W. Date and J. R. Singham, Numerical prediction of friction and heat transfer characteristics of fully developed laminar flow in tubes containing twisted tapes, ASME paper 72-HT-17 (1972).
5. A. W. Date, Prediction of fully-developed flow in a tube containing a twisted-tape, *Int. J. Heat Mass Transfer* **17**, 845 (1974).
6. S. W. Hong and A. E. Bergles, Augmentation of laminar flow heat transfer in tubes by means of twisted-tape inserts, Iowa State University Heat Transfer Laboratory Report HTL-5, ISU-ERI-Ames-75011 (December 1974).
7. A. W. Date, Personal communication (23 July 1975).

*The results given in [5] are in error by a factor of 2·0 [7]. The numbers given here incorporate this correction.